Images from the Hubble Space Telescope suffer from an overcorrected spherical aberration that is due to a conic-constant error in the primary mirror. Within the program known as the corrective optics space telescope axial replacement (COSTAR), simulators have been built to provide the point-spread function (PSF) of the telescope alone and of the telescope with the faint-object camera F/96. It was found that the experimental PSF's were identical to those in orbit, which was not the case when the PSF's were calculated with commonly used optical software. We explain this discrepancy and propose a modeling method that is based on the determination of the wave-front error at the exit-pupil level that gives results that are consistent with observations.

Key words: Hubble Space Telescope, optical path difference, point-spread-function modeling, wave-front error.

1. Introduction

In late June 1990, a few weeks after the launch of the Hubble Space Telescope (HST), scientists observed that the images being received were not as expected: approximately 16–18% of the light from a point source was concentrated within a 0.1″ radius instead of the 70% of light that was expected. A wave-front error that could not be corrected by adjustment of the alignment of the optics (e.g., refocus or secondary-mirror tilt) was present in the optical system. The analysis by wave-front reconstruction of in-flight images, which were obtained either with the faint-object camera F/96 (FOC96) or with the wide-field planetary camera, revealed a strong spherical aberration that is due to a conic-constant error in the telescope's primary mirror. This mirror was found to have been polished too flat and therefore was overcorrected by approximately 2 µm at the edge when compared with the ideal figure.

To fully recover the HST scientific performance, a special study group, the HST strategy panel, recommended the development of the corrective-optics space telescope axial replacement (COSTAR), which consists of five pairs of small, corrective mirrors that are mounted on a deployable optical bench. The bench is inside a dedicated new axial instrument, which is to be installed in the HST in place of the high-speed photometer. Two simulators of the aberrated HST were built to permit full verification of the COSTAR performance during its development in 1991–1993: the refractive aberrated simulator (RAS) by Ball Aerospace and the optical simulator Laboratoire d'Astronomie Spatiale (OSL) by the European Space Agency. In addition, an existing structural thermal model of the FOC96 (FOC/STM) was refurbished so as to be identical to the actual FOC96. The COSTAR was successfully installed in the HST during the 12–13 December 1993 first-servicing mission, and it provided full recovery of the image quality.

The simulators and the FOC/STM provide several optical configurations: 1) the aberrated telescope alone (RAS or OSL); 2) the nominal telescope alone (OSL); 3) the aberrated telescope with the FOC96 (RAS or OSL followed by the FOC/STM F/96); 4) RAS with COSTAR; and 5) RAS with COSTAR and FOC/STM. Configuration 2 was used to check that the FOC/STM was properly aligned, whereas configurations 3–5 were employed to provide images identical to those in-flight and to verify the corrective performance of the COSTAR optics and its in-flight alignment capability. As expected, the diffraction images obtained with configurations 1 and 3, when observed at the same scale, were identical, because the FOC96 has been designed to be stigmatic.
and therefore to introduce no additional aberration\textsuperscript{13} except for the slight correction of the telescope's residual astigmatism of 0.16\textmu{}rms at the best focus,\textsuperscript{6} which has to be taken into account but does not affect the results of this study. This result also confirms that the aberrations that affect the in-flight images were due only to the telescope itself.

However, when the diffraction images for the configurations 1 and 3 are calculated by means of the commercially used optical design software, LASSO (an in-house optical program, CODE V, GENESEE, SYNOPSIS, and others), the software warns that the results may be erroneous because of the large aberrations of CODE V or the images are different for the two configurations. As shown in Figs. 1 and 2 (SYNOPSIS output), the diffraction image for the telescope with FOC96 is better than the image obtained for the telescope alone. This result, which disagrees with the observed experimental images, leads us to conclude that these standard models are not well suited to our problem of modeling a large spherical aberration that is magnified by a stigmatic camera.

The aim of our present study is to explain this discrepancy and to propose a model that gives results in agreement with the experimental images. We first briefly recall the concepts commonly used to describe geometrical aberrations—the optical path difference (OPD) and the wave-front error (WFE)—and the classical method of calculating the point-source image of an optical system—the point-spread function (PSF). The calculation of the OPD and the WFE in the third-order approximation permits light to be shed on the problem. Then we show that only the use of the WFE leads to a correctly aberrated PSF of the HST with FOC96. Finally, we calculate the PSF's of configurations 1 and 3 (the HST alone and the HST with the FOC96, respectively) using the WFE method, and we compare those PSF's with the diffraction images obtained in flight and in experimental simulations with the simulators (RAS or OSL) and the FOC/STM.

2. Optical Path Difference, Wave-Front Error, and Point-Spread Function

A. Definitions

When an optical system has no aberrations, a point image A' corresponds to a point source A (Fig. 3). According to the principle of Fermat, the optical length AA' is constant and does not depend on the particular ray traced between A and A'.\textsuperscript{12} In this case, the wave fronts in the image space are spheres centered on the Gaussian image A' of the object A. Let us call the reference sphere, S_{ref}, the sphere that is centered at A' and contains the exit-pupil center w'. For an aberrated system, the ray from A in Fig. 3 does not go to A' because the actual wave front, S_{real}, is distorted. The ray that comes from A passes through S_{real} at M\textsubscript{1}, through S_{ref} at M, and crosses the paraxial image plane at A\textsubscript{1}'. The unaberrated optical path (AA') is followed only by the principal ray, Aww'A'. We usually define the aberrant optical path as AH, where H is the foot of the perpendicular from A to the actual ray.\textsuperscript{13} The difference between optical paths (AH) and (AA') is called the OPD. The optical length (MM\textsubscript{1}) between S_{ref} and S_{real} is called the
normal distance\textsuperscript{14} or the WFE. Because $M_1$ and $w'$ are part of $S_{\text{real}}$, the WFE $\langle MM_1 \rangle$ is given by

\begin{equation}
\langle MM_1 \rangle = \langle AM \rangle - \langle AM_1 \rangle = \langle AM \rangle - \langle Aw' \rangle.
\end{equation}

B. Relation between the OPD and the WFE
According to the definitions

\begin{align*}
\text{OPD} &= \langle AH \rangle - \langle AA' \rangle \\
&= \langle AM_1 \rangle + \langle M_1 M \rangle + \langle MH \rangle - \langle Aw' \rangle - \langle w'A' \rangle.
\end{align*}

Because $w'$ and $M_1$ are part of $S_{\text{real}}$ and $w'$ and $M$ are part of $S_{\text{ref}}$, we have

\begin{align*}
\langle AM_1 \rangle &= \langle Aw' \rangle \\
\langle w'A' \rangle &= \langle MA' \rangle.
\end{align*}

Then

\begin{equation}
\text{OPD} = \langle M_1 M \rangle + \langle MH \rangle - \langle MA' \rangle
\end{equation}

\begin{equation}
= \text{WFE} + \langle MH \rangle - \langle MA' \rangle.
\end{equation}

Figure 3 shows that

\begin{equation}
\text{OPD} - \text{WFE} = \langle MH \rangle - \langle MA' \rangle = R' \langle \cos d\alpha' \rangle - 1,
\end{equation}

and we obtain to the first order

\begin{equation}
\text{OPD} - \text{WFE} \approx R' \frac{\langle d\alpha' \rangle^2}{2} = \frac{\langle dy' \rangle^2}{2R'},
\end{equation}

where $dy'$ represents the transverse-ray aberration,\textsuperscript{15} $d\alpha'$ represents the angular-ray aberration, and $R'$ represents the radius of the reference sphere. In the most general case, the OPD and the WFE are not equal. However, when $d\alpha'$ is small with respect to $\alpha'$ or when $dy'$ is small with respect to $R'$, the radius of the reference sphere, we can consider that the OPD is a very good approximation to the WFE. We will show that this approximation is valid for calculating the aberrated PSF of the HST alone but that it is not valid for calculating the PSF of the HST with the FOC96.

C. PSF
The PSF is the image of a point source at infinity in our case and corresponds to the square of the absolute value of the diffracted amplitude. The encircled energy is the total energy inside a circle of a given radius; it is generally expressed in arcseconds. If we use polar coordinates which are more appropriate for a circular aperture and apply the Huygens-Fresnel principle,\textsuperscript{16} the diffracted amplitude at a point $P(r_1, \theta_1)$ in the focal plane is given by

\begin{equation}
U(r_1) = \frac{\exp[jk]}{j\lambda} \frac{\exp(jk)}{2\pi r_1^{*}}
\end{equation}

\begin{equation}
\times \int_{R_0}^{R} 2\pi \exp \left[ \frac{2\pi}{\lambda} \Phi(r_0) \right] \frac{k}{f} dr_0 dr_0,
\end{equation}

where $k$ denotes the wave vector, $f$ is the focal length of the optical system, $\lambda$ is the wavelength, $\Phi_0$ is the aberration function,\textsuperscript{17} $J_0$ is the zero-order Bessel function of the first kind, and $R_0$ and $R$ are the inner and outer radii, respectively, of the aperture. The aberration function is equal to the WFE, but several optical software products (GENESEE and SYNOPSIS, for example) use the OPD as the aberration function.

3. Third-Order Study
In the case of the HST, we can expect that the third-order approximation correctly represents the optical design because the aperture angle and the field angle are small (paraxial field).

A. WFE Resulting from the Primary Mirror
The figuring profile of the primary mirror is

\begin{equation}
z = \frac{H^2}{2R_c} + \left[1 + \varepsilon \right] \frac{H^4}{8R_c^3},
\end{equation}

where $R_c$ is the radius of curvature, $H$ is the height of the ray on the primary mirror, and $\varepsilon$ is the conic constant ($\varepsilon = 0$ for a sphere, and $\varepsilon = -1$ for a parabola). The mirror-fitting error is then

\begin{equation}
\Delta z = \frac{\Delta \varepsilon}{8R_c^3} H^4,
\end{equation}

so the corresponding WFE that is due to the primary mirror is

\begin{equation}
\text{WFE} = 2\Delta z = \frac{\Delta \varepsilon}{4R_c^3} H^4.
\end{equation}

B. Difference (OPD - WFE) at the Exit Pupil of the Telescope
In the third-order approximation, we may add the WFE contribution of the different components of an optical system.\textsuperscript{18} As the primary mirror is the entrance pupil and assuming that its conic-constant error and some defocus are the only contributions, we find that the WFE in the telescope image space is

\begin{equation}
\text{WFE} = \frac{df}{2} + \frac{\Delta \varepsilon}{4R_c^3} H^4 = \frac{df}{2} + \frac{f^4 \Delta \varepsilon}{4R_c^3} \alpha'^4,
\end{equation}

where $H = f \alpha'$, $f$ denotes the telescope focal length, $\alpha'$ is the slope angle of the image marginal ray, and $df$ is
the distance from the paraxial focus to the actual focal plane.
The corresponding transverse aberration dy’ is

\[
dy'_{\text{HST}} = \frac{\partial \text{WFE}[\alpha']}{\partial \alpha' \cos \alpha'} = d_1 \alpha' + \frac{f^4 \Delta c}{R_c^3} \alpha'^3. \tag{12}
\]

According to relations (6) and (12), we have

\[
\langle \text{OPD} - \text{WFE} \rangle_{\text{HST}} \approx \frac{(dy'_{\text{HST}})^2}{2R_{\text{HST}}} \approx \frac{1}{2R_{\text{HST}}}
\times \left( d_{1\text{HST}} \alpha' + \frac{f_{1\text{HST}}^4 \Delta c}{R_c^3} \alpha'^3 \right)^2,
\tag{13}
\]

where \( R_{\text{HST}} \) is the distance from the paraxial focus to the telescope exit pupil.

C. Difference \( \langle \text{OPD} - \text{WFE} \rangle \) at the Exit Pupil of the FOC96
Because the FOC96 is stigmatic, the WFE has the same value, i.e., the FOC96 provides a magnification \( m \) and \( \alpha' \) becomes \( \alpha'/m \), \( f \) becomes \( mf \), \( df \) becomes \( m^2 df \), and \( dy' \) becomes \( m dy' \). According to relation (6) we have

\[
\langle \text{OPD} - \text{WFE} \rangle_{\text{FOC96}} \approx \frac{m^2 (dy'_{\text{HST}})^2}{2R_{\text{FOC96}}} \approx \frac{m^2}{2R_{\text{HST}}}
\times \left( d_{1\text{HST}} \alpha' + \frac{f_{1\text{HST}}^4 \Delta c}{R_c^3} \alpha'^3 \right)^2.
\tag{14}
\]

Hence

\[
\langle \text{OPD} - \text{WFE} \rangle_{\text{FOC96}} \approx \frac{m^2 R_{\text{HST}}}{R_{\text{FOC96}}} \langle \text{OPD} - \text{WFE} \rangle_{\text{HST}}. \tag{15}
\]

4. Results
Let us assume the following numerical values: \( \Delta c = 0.0116 \), \( R_c = 11,041 \) mm, \( f_{\text{HST}} = 57,600 \) mm, \( R_{\text{HST}} = 7003 \) mm, \( R_{\text{FOC96}} = 1859 \) mm, \( m = 4 \), and \( d_{1\text{HST}} = 13.5 \) mm. This value of \( d_{1\text{HST}} \) is the defocus for the best

focus of the telescope as determined in flight through an optimization of the encircled energy at 0.1 arcsec of the FOC96 images.

Table 1 gives the results of the third-order approximation described above. In the telescope’s focal plane, the difference between the OPD and the WFE is small compared with the aberration. So we can use both of them to characterize the PSF. On the contrary, at the FOC96 image plane, the difference is of the same order as the aberration, and the OPD cannot be used to model the PSF.

A. HST Images
These results are confirmed in Table 2 through the use of our in-house software. For this configuration, the OPD and the WFE are very close: both methods agree because the aberrations are small compared with the radius of the reference sphere, the longitudinal spherical aberration equals 41 mm, whereas the radius of the reference sphere equals 7003 mm.

B. HST and FOC96 Images
To study the spherical aberration, which is rotationally symmetric, we can assume that the telescope and the faint-object camera are on-axis. Although in the actual design, the faint-object camera is slightly off-axis, this assumption is valid because the telescope astigmatism is corrected inside the FOC96 by

<table>
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<tr>
<th>Height (mm)</th>
<th>SYNOPSIS OPD_{HST} [\mu m]</th>
<th>SYNOPSIS WFE_{HST} [\mu m]</th>
<th>In-House Program OPD_{HST} [\mu m]</th>
<th>In-House Program WFE_{HST} [\mu m]</th>
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<td>-0.270172</td>
<td>-0.270172</td>
<td></td>
</tr>
</tbody>
</table>

*The calculations are done for the nominal conic-constant error 0.0116 and at the best focus 13.5 mm from the paraxial focus.
means of a folding cylindrical mirror. The only remaining significant aberration is the spherical one that is due to the primary mirror.

Table 3 shows that, near the optical axis (i.e., for incidence heights less than 720 mm), the WFE's are very close to the OPD: the difference is of the order of 1 nm, and the aberrations remain small. On the other hand, at the pupil edges, the WFE is up to 11 times larger than the OPD; thus, in this area, only the WFE correctly characterizes the spherical aberration, because the aberrations are no longer negligible with respect to the radius of the reference sphere. The longitudinal spherical aberration equals 656 mm, and the radius of the reference sphere equals 1859 mm.

Comparing Tables 2 and 3 we find that the WFE of the two configurations 1 and 3 for all incidence heights are equal to a good approximation. This result is consistent with the experiments and shows that the use of the WFE is the only correct approach to model this specific case of large spherical aberrations.

The diffracted intensity in the focal plane and the encircled energy at 488 nm are illustrated in Figs. 4 and 5 for configurations of the HST with and without the FOC96, respectively. In both cases the diffraction images are the same. This theoretical result is in full agreement with the observations performed with the simulators. Finally, a comparison between the PSF observed in flight and that obtained with our model (Fig. 6) shows that our method describes the as-built HST well. Moreover, this comparison gives additional confidence in the adopted value for the as-built HST primary-mirror conic constant.

5. Conclusion
The use of the WFE accurately describes the aberrated diffracted image of the Hubble Space Telescope with the faint-object camera F/96. The comparison between this method and the usual one, which is based on the use of the OPD, shows that the OPD method is not applicable to the present design. This study shows that, for a strongly aberrated system, the WFE method efficiently characterizes the PSF, whereas the OPD method does not. However this latter gives a correct PSF for optical systems whose aberrations are small with respect to the radius of the reference sphere, which is generally the case, and that is why the OPD method is commonly used in optical software products.

The PSF computed with the WFE method matches well both the PSF observed in flight with the FOC96 and the PSF image given by the simulators with the FOC/STM. This study should contribute to a better understanding of the actual optical system of the Hubble Space Telescope and provide more confidence in the knowledge of the conic-constant error and in the potential of the COSTAR program to correct the error. This capability has been confirmed by the quality of the full-image recovery that has been obtained with the COSTAR since January 1994.

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